EFFECT OF THERMOCAPILLARITY ON PARAMETRIC EXCITATION OF SURFACE WAVES

V. G. Nevolin

UDC 532.516.501.34:532.68

It is demonstrated that the thermocapillarity effect can either raise or lower the threshold of surface wave excitation, depending on the conditions of heating.

Under consideration is a horizontal layer of fluid with a free surface, bounded from below by an isothermal solid plate held at a constant temperature $\gamma\vartheta^0$ where $\gamma = 1$ in the case of heating from below and $\gamma = -1$ in the case of heating from above. The coefficient of surface tension σ depends on the temperature ϑ according to the relation $\alpha = \alpha_0 - \sigma\vartheta$ $(\sigma > 0)$.

The corresponding Cartesian system of coordinates is selected so that the xOy plane will coincide with the unperturbed interface and with the z axis oriented vertically upward. The layer moves about the vertical axis according to the law $a \cos \omega t$ with an amplitude $a < 1$. The fluid is assumed to be incompressible. In the reference system tied to the layer, we write the equations of motion conventionally [1], but replace the gravitational acceleration $\mathbf{g} = (0, 0, -g)$ with $\mathbf{g}(t) = (1 - \eta \cos \omega t)\mathbf{g}$, where $\eta \equiv a\omega^2/g$.

Inasmuch as oscillatory motion of the fluid layer does not disturb the equilibrium possible in this system but only modulates the pressure, the conditions for equilibrium become

$$\mathbf{v}_0 = 0, \quad \vartheta_0 = -Gz, \quad \zeta_0 = 0, \quad G \equiv \gamma\vartheta^0/h,$$
$$p_0 = -\rho g (1 - \eta \cos \omega t)(z + \beta G z^2/2). \tag{1}$$

Here both pressure p and temperature ϑ are read from the surface as datum.

1. Let us examine the stability of this equilibrium, for the purpose of which we will, as usually, analyze perturbations of velocity, pressure, and temperature [1]. We select $(\alpha_0/\rho g)^{1/2}$, $(\alpha_0/\rho g^3)^{1/4}$, $(\rho g^3/\alpha_0)^{1/4}$, $(\alpha_0 g/\rho)^{1/4}$, $(\alpha_0 \rho g)^{1/2}$, and $G(\alpha_0/\rho g)^{1/2}$ as units of length, time, frequency, velocity, pressure, and temperature, respectively, so that for the perturbations we obtain the linearized system of equations

$$\frac{\partial \mathbf{v}}{\partial t} = -\nabla p + \frac{1}{A}\nabla^2 \mathbf{v} - q\vartheta \mathbf{g}(t)/g,$$
$$P\left(\frac{\partial \vartheta}{\partial t} - w\right) = \frac{1}{A}\nabla^2\vartheta, \quad \nabla \cdot \mathbf{v} = 0, \tag{2}$$

where $A \equiv \nu^{-1}(\alpha_0^3/\rho^3 g)^{1/4}$; $q \equiv \beta G(\alpha_0/\rho g)^{1/2}$; $P \equiv \nu/\chi$.

For a small deviation ζ of the surface from its equilibrium position we obtain at $z = 0$ [1, 2]

$$\frac{\partial \zeta}{\partial t} = w, \quad \vartheta - \zeta + \frac{1}{b}\frac{\partial \vartheta}{\partial z} = 0,$$
$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = -\mu A\left(\frac{\partial \vartheta}{\partial x} - \frac{\partial \zeta}{\partial x}\right),$$
$$\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = -\mu A\left(\frac{\partial \vartheta}{\partial y} - \frac{\partial \zeta}{\partial y}\right), \tag{3}$$
$$p = \left[1 - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) - \eta \cos \Omega t\right]\zeta + \frac{2}{A}\frac{\partial w}{\partial z},$$

where $\mu \equiv \gamma\vartheta^0\sigma/\alpha_0 H$; $b \equiv l(\alpha_0/\rho g)^{1/2}/\varkappa$. It follows from the definition of μ that $\mu > 0$ in the case of heating from below and $\mu < 0$ in the case of heating from above.

Perm State University. Design Institute of Petroleum Industry. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 46, No. 3, pp. 424-428, March, 1984. Original article submitted November 23, 1982.

0022-0841/84/4603-0303$08.50

At the bottom of the layer $z = -H$ ($H \equiv h/h_1$) we have

$$\mathbf{v} = 0, \quad \vartheta = 0. \tag{4}$$

For simplicity, we will consider the case of a layer thickness h larger than the capillary thickness $h_1 \equiv (\alpha_0/\rho g)^{1/2}$ but smaller than the thermocapillary depth $h_2 \equiv (\sigma/\rho g\beta)^{1/2}$ or $1 \ll H < h_2/h_1 = (\sigma/\alpha_0\beta)^{1/2}$. This will allow us, on the one hand, to consider an infinitely deep layer and, on the other hand, to disregard in the system of equations (2) the term with the convective lifting force [2, 3], which is valid for sufficiently thin layers of fluid. These assumptions are not contradictory, since for most fluids $h_2/h_1 \gg 1$ and the penetration depth for waves is approximately equal to h_1.

We perform the Fourier transformation with respect to variables x and y. Considering that the quantities $|\mathbf{v}|$, $|\zeta|$, $|\partial \mathbf{v}/\partial x_j|$, $|\partial\zeta/\partial x_j|$ are bounded at $|x_j = x, y| \to \infty$ and eliminating the pressure as well as the x, y components of velocity, we obtain instead of system (2)-(4)

$$\left(\frac{\partial^2}{\partial z^2} - k^2\right)^2 w = A\left(\frac{\partial^2}{\partial z^2} - k^2\right)\frac{\partial w}{\partial t},$$

$$\left(\frac{\partial^2}{\partial z^2} - k^2\right)\vartheta - A\,\mathrm{P}\frac{\partial\vartheta}{\partial t} = -\mathrm{P}Aw, \quad k^2 \equiv k_x^2 + k_y^2. \tag{5}$$

At $z = 0$ we have

$$\frac{\partial\zeta}{\partial t} = w, \quad \vartheta - \zeta + \frac{1}{b}\frac{\partial\vartheta}{\partial z} = 0, \quad \frac{\partial^2 w}{\partial z^2} + k^2 w + k^2\mu A(\vartheta - \zeta) = 0, \tag{6}$$

$$\frac{1}{k}\left[\frac{\partial}{\partial t} - \frac{1}{A}\left(\frac{\partial^2}{\partial z^2} - 3k^2\right)\right]\frac{\partial w}{\partial z} + (\Omega_0^2 - \varphi\cos\Omega t)\,\zeta = 0, \tag{7}$$

where $\mathbf{k} = (k_x, k_y)$ is the wave vector, $\Omega^2 \equiv k^3 + k$, and $\varphi \equiv \eta k$.

The general solution to problem (5) with conditions (4), (5), (7) consists of solutions with nonhomogeneous boundary conditions ($\zeta \neq 0$) and with homogeneous ones ($\zeta = 0$), respectively.

Solving Eqs. (5) for $\zeta = 0$, we obtain for $w(t)$ and $\vartheta(t)$

$$w = c^0(\exp kz - \exp\sqrt{k^2 + A\lambda}\,z)\exp\lambda t, \quad \vartheta = c^0(\lambda^{-1}\exp kz$$
$$+ c_1^0\exp\sqrt{k^2 + \mathrm{P}A\lambda}\,z - [\mathrm{P}/(\mathrm{P}-1)\lambda]\exp\sqrt{k^2 + A\lambda}\,z\}\exp\lambda t, \tag{8}$$

where λ is determined from the equation $\lambda^2(b + \sqrt{k^2 + \mathrm{P}A\lambda}) = k^2\mu[(\sqrt{k^2 + A\lambda\mathrm{P}} - \sqrt{k^2 + \mathrm{P}A\lambda})/(\mathrm{P} - 1) - k]$. Coefficient c_1^0 is determined from conditions (6) with $\zeta \equiv 0$.

Solving Eqs. (5) for nonhomogeneous boundary conditions by the method of Laplace transformation with respect to time and letting $\zeta(t = 0) = w(t = 0) = \vartheta(t = 0) = 0$, we obtain

$$W(s) = [sZ(s) - C(s)]\exp kz + C(s)\exp\sqrt{k^2 + As}\,z,$$

where

$$C(s) = -2k^2 Z(s)/A + \mu k^3 Z(s)/s(b + \sqrt{k^2 + \mathrm{P}As}) + \tag{9}$$
$$+ 2k^4\mu\left(k + \frac{\sqrt{k^2 + \mathrm{P}As} - \mathrm{P}\sqrt{k^2 + As}}{\mathrm{P} - 1}\right)Z(s)/A^2 s^2(b + \sqrt{k^2 + \mathrm{P}As}).$$

Here $W(s)$ and $Z(s)$ are the Fourier transforms of functions $w(t)$ and $\zeta(t)$, respectively.

2. We now insert expression (9) into Eq. (7) and perform the inverse Laplace transformation. Using the solution (8) to the homogeneous boundary-value problem with accuracy $1/A$ and $\mu/\sqrt{A}$, for finite values of the heat transfer coefficient b we obtain for $\zeta(t)$ the equation

$$\frac{d^2\zeta}{dt^2} + 2\delta\frac{d\zeta}{dt} - \varepsilon\int_0^t \zeta(t-\tau)\frac{d\tau}{\sqrt{\pi\tau}} + (\Omega_0^2 - \varphi\cos\Omega t)\,\zeta = -c^0(\lambda + \delta - \delta\sqrt{1 + 2\lambda/\delta})\exp\lambda t \equiv f(\lambda, t), \tag{10}$$

where $\delta \equiv 2k^2/A$, $\varepsilon \equiv \mu k^3/\sqrt{A\mathrm{P}}$.

It follows from Eq. (10) that in this approximation and with $\lambda(b) = 0$ the stability of equilibrium of the fluid surface does not depend on the conditions of heat transfer.

In the case of large b the equation of motion for ζ will be analogous, but it will be necessary to let $\varepsilon = 0$ and to reduce Ω_0^2 by $\mu k^3/b$.

When $\varepsilon = 0$, then Eq. (10) is a nonhomogeneous Mathieu equation with damping. Depending on the conditions of heating, function $f(\lambda, t)$ can increase in time when $\mu > 8k(k + b)/PA^2$ or decrease in time when $\mu < 8k(k + b)/PA^2$. In the case of steady convection we have $\lambda = 0$ and $f(\lambda, t)$ becomes zero.

When $\lambda > 0$, then on the surface of the fluid can appear waves even in the absence of parametric excitation. Indeed, a solution of Eq. (10) with $\varphi = 0$ by the method of averaging yields [4, 5]

$$\zeta(t) = \left\{\int_0^t f(\lambda, \tau)\sin\Omega_0(t-\tau)\frac{d\tau}{\Omega_0^2} - M_1\cos\Omega_0 t + M_2\sin\Omega_0 t\right\}\exp(-\delta t) + \left\{M_1\cos\left(\Omega_0 - \frac{\varepsilon}{\sqrt{8\Omega_0^3}}\right)t \right. \tag{11}$$

$$\left. - \frac{M_1\varepsilon/\sqrt{2\Omega_0} - 2M_2\Omega_0^2}{\varepsilon/\sqrt{2\Omega_0} - 2\Omega_0^2}\sin\left(\Omega_0 - \frac{\varepsilon}{\sqrt{8\Omega_0^3}}\right)t\right\}\exp\left(-\delta - \frac{\varepsilon}{\sqrt{8\Omega_0^3}}\right)t,$$

$$M_1 = (D_0 + D_1)\sqrt{2/\Omega_0}, \quad M_2 = (D_0 - D_1)\sqrt{2/\Omega_0},$$

$$\begin{pmatrix} D_0 \\ D_1 \end{pmatrix} = \lim_{T\to\infty}\frac{1}{T}\int_0^T\begin{pmatrix}\cos\Omega_0 t \\ \sin\Omega_0 t\end{pmatrix}\left[\int_0^t\frac{d\tau}{\sqrt{\pi(t-\tau)}}\left(\int_0^\tau f(\lambda, \varkappa)\sin\Omega_0(\tau-\varkappa)\,d\varkappa\right)\right]dt.$$

This indicates that a wave will appear on the surface of the fluid even without parametric excitation if

$$\lambda \geqslant \delta, \ \varepsilon > 0; \ \lambda \geqslant \delta + \varepsilon/(8\Omega_0^3)^{1/2}, \ \varepsilon > 0. \tag{12}$$

In the case of heating from above ($\varepsilon < 0$), surface waves will appear on the surface of the fluid at lower values of λ. When b is large, then excitation of waves occurs at $\lambda \geqslant \delta$.

When $\varphi \neq 0$ and $\lambda \leqslant 0$ ($\mu \leqslant 8k(k + b)/PA^2$), then on the surface appear parametric waves with frequency $\Omega/2$ at

$$\varphi \geqslant \varphi_* = \frac{4k_*\Omega}{A}\left(1 + \frac{\mu k_*}{2}\sqrt{\frac{A}{P\Omega^3}}\right), \tag{13}$$

with k_* determined from the equation

$$k_*^3 + k_* = \Omega^2/4. \tag{14}$$

The expression for function $\zeta(t)$ here is analogous to expression (11), with $\Omega/2$ replacing Ω_0 and $\varphi/2\Omega - 2k(1 + \mu k\sqrt{A}/4P\Omega^3)/A$ replacing $(-\delta - \varepsilon/\sqrt{8\Omega_0^3})$.

It follows from relation (14) that by controlling the conditions of heating (from below or from above) one can either inhibit or facilitate parametric excitation of surface waves.

When $\varphi \neq 0$ and $\lambda > 0$, then parametric excitation of surface waves occurs at $\varphi \geqslant \varphi_*$ if $\lambda \leqslant \varphi/2\Omega - (\delta + \varepsilon/\sqrt{2\Omega^3})$. When $\lambda > \varphi/2\Omega - (\delta + \varepsilon/\sqrt{2\Omega^3})$, then on the surface of the fluid appears a wave with frequency $\Omega/2$ in the case of resonance (14) or with frequency Ω otherwise. In the latter case expression (13) defines the conditions for excitation of waves with frequency Ω.

NOTATION

α, coefficient of surface tension; ν, kinematic viscosity; ω, angular velocity; Ω, dimensionless frequency; ρ, density of the fluid; β, coefficient of thermal volume expansion; χ, thermal diffusivity; η, dimensionless modulation amplitude; σ, thermocapillary constant; ϑ^0, ϑ_0, ϑ, temperatures; δ and ε, dissipation parameters; μ, a parameter; ζ, deviation of the surface from its equilibrium position; $\varkappa$, thermal conductivity; λ, increment; x, y, z, Cartesian coordinates; t, time; a, vibration amplitude; g, gravitational acceleration; $\mathbf{v} = (u, v, w)$, velocity vector; p_0, p, pressures; G, temperature gradient; P, Prandtl number; 1/A, a small parameter; k_j, wave number along the j-th axis; W(s), Fourier transform of the surface deviation from the equilibrium position; Z(s), Fourier transform of the z-component of velocity; h, dimensional layer thickness; H, dimensionless layer thickness; b, dimensionless heat-transfer coefficient; l, heat-transfer coefficient; and c^0, normalizing factor.

LITERATURE CITED

1. L. D. Landau and E. M. Lifshits, Mechanics of Continuous Media [in Russian], GITTL, Moscow–Leningrad (1944).
2. G. Z. Gershuni and E. M. Zhukovitskii, Convective Stability of Incompressible Fluids, Halsted Press (1976).
3. R. V. Birikh, "Thermocapillary convection in horizontal layer of fluid," Zh. Prikl. Mekh. Tekh. Fiz., No. 3, 69-72 (1966).
4. A. N. Filatov, Method of Averaging for Differential and Integrodifferential Equations [in Russian], FAN, Tashkent (1971).
5. V. V. Moskvitin, Resistance of Viscoelastic Materials [in Russian], Nauka, Moscow (1972).

CONTACT HEAT AND MASS TRANSFER BETWEEN A HOT GAS AND A FLOWING LIQUID FILM

P. I. Geshev, O. P. Kovalev,
O. Yu. Tsvelodub, and Yu. V. Yakubovskii

UDC 536.27.001.572

A mathematical model of contact heat and mass transfer is proposed and the calculations are compared with the experimental data.

One method of utilizing the heat of the flue gases escaping from thermal power plants is to employ them to heat water in film contact apparatus; this has less resistance than a packed column, so that the apparatus can be mounted in the exhaust system of the power plant without it being necessary to install an exhaust fan.

In order to investigate the process of heat and mass transfer in heaters of this kind we designed an experimental set-up [1] in which the film contact apparatus consisted of three modules comprising box-type structures of rectangular cross section measuring 0.5 × 0.3 × 1.0 m with three working plates 0.5 m wide and 0.94 m high, the distance between plates being 0.05 m. In order to eliminate heat losses to the ambient medium, all the modules were insulated on the outside with asbestos lagging.

The liquid film, created by special sprinklers [2], flowed over both sides of the plates and the inner surfaces of the module housing, so that the gas was able to pass through four channels with wetted walls.

The mean temperature of the liquid was measured with thermocouples mounted in the water headers, at the inlet to the equipment, between the modules and at the outlet. The gas temperature was measured at the inlet to the equipment and at the outlet. The moisture content of the gas at the inlet was calculated with allowance for the humidity of the starting air and the moisture formed as a result of combustion of the fuel. The flow rates of fuel and water were measured volumetrically, that of the starting air from the pressure drop in a convergent channel. The final moisture content of the gas was determined by the balance method.

Direct contact between the hot gas and the liquid film is accompanied by simultaneous processes of convective heat transfer in the film and the gas phase and diffusion of vapor in the gas. These processes are interrelated and rather difficult to calculate.

We solved this problem by means of a contact heat and mass transfer model based on averaged differential heat and mass transfer equations, subject to the following assumptions: by virtue of symmetry the heat fluxes at a wetted plate of infinitely small thickness and at the channel axis are equal to zero, the transfer processes are steady-state, the motion of the gas and the liquid is directed along the x axis, there are no heat losses to the ambient medium, changes of flow rates due to phase transitions can be neglected.

Institute of Thermophysics, Siberian Branch, Academy of Sciences of the USSR, Novosibirsk. Far Eastern Polytechnic Institute, Vladivostok.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 46, No. 3, pp. 428-432, March, 1984. Original article submitted December 7, 1982.

0022-0841/84/4603-0306$08.50